On the Probabilities of a waitingtime between the Landworkers and Tractor-transportations in the Landing and Loggingwork.

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I) Introduction

The logging work with a tractor is divided into three steps ;

1) works in the place of felling
2) hauling with a tractor
3) works in the landing.

The kinds of works of each step are different according to the ways of hauling timbers; as the whole tree length or as logs. For example, in case of the former, the works in the felling place are felling and hooking, whereas in case of the latter felling, hookin and conversion (making logs) as well.

Let us consider about the waiting states of the workers in a landing and a tractor.

When a load of timbers is brought to a landing, if any workers are there who are not engaged with other works, it will be disposed at once, but if there are already other timbers and all the workers are engaged, it will be left there and wait its turn of disposal. And when the landing has been filled with timbers which has not yet been disposed.... I call this undisposed timber hereafter in this text...., the tractor cannot continue the work any more and be obliged to get into the state of waiting. On the other hand, when there are not any undisposed timbers in the landing, all the workers are in a waiting state, though this is not always happening.

Here comes a problem ; what is the probability that each state of waiting occurs ? I shall deal with this problem in this text. I take the theory of Stochastic Processes for this problem.

II) The preparation from the theory by D. G. Kendall.

This section is a forest version from Kendall's papers ; "Some Problems in the Theory of Queues". (in Journal of the Royal Statistical Society, series B, Vol. XIII, No. 2, 1951)

First of all, I must describe several assumptions for the theory of D. G. Kendall.

It is here assumed that all the workers in the landing must be engaged in disposeing of every load of timbers at once without exception, so nobody touches any other loads during the disposal.

Considering the matter from the viewpoint of Macro, this assumption is not conflicted with the actual.

Suppose that each load brought to the landing is disposed in the order of their arrivals, and there will be a quite number of loads waiting their turns.

Here, let E_n represent for the state of the landing with the n loads of timbers including the one being disposed. And then E_o is for the empty landing when all the workers are waiting, and E_1 for the state with only one load being disposed, and so on.

The state E_n is changes all the time. If it can be supposed that the arrivals of tractors belong to the distribution of Poissontype, it is possible to take the fluctuations of E_n as an enumerable Markovian chain if the attention is directed to the epochs at which the disposal of each load is finished (Kendall and Bartlett called these epochs regeneration points).

When the hypothesis that a tractor arrives at random holds, then the number of arrivals in the time t is a Poisson variability of expectation t/a, say :

$$P_r = \frac{\left(\frac{t}{a}\right)^r e^{-\frac{t}{a}}}{r!}$$

(P_r : The Probability of arrivals γ in the time t)

Note 1):

A sequence of E_o, $E_1 \ldots.$, $E_n \ldots.$, is called a Markovian chain, if the probabilities of the states E_m (m=0, 1....,) occur at time t+h when the states E_n (n=0, 1....,) at arbitrary time t_o depnd only on E and have not any relations to the states at any time $t<t_o$.

Now let a $\left(=\frac{1}{\lambda}\right)$ represent for the expectation of the timeinterval u between two consecutive arrivals of tractors, and b for the expectation of v which denotes the time which elapses while a certain load is being disposed ; I call this v the disposing time or the service time. And assume that v has a distributing function B (v).

Then the probability k_r of r arrivals in the time v is

$$K_r = \frac{1}{r!}\int_0^\infty e^{-\frac{v}{a}}\left(\frac{v}{a}\right)^r d\beta(v) \cdots\cdots(1)$$

These k_r's (r=0, 1....,) are the probability distribution, being $\sum k_r = 1$.

Let P_{ij} represent for the transition-probability from E_i to E_j, and the transition matrix (P_{ij}) is

$i \diagdown j$	0	1	2	3…
0	k_0	k_1	k_2	$k_3\cdots$
1	k_0	k_1	k_2	$k_3\cdots$
2	0	k_0	k_1	$k_2\cdots$
3	0	0	k_0	$k_1\cdots$
	⋮	⋮	⋮	⋮

From the theorems of the theory of Stochastic Processes, the probabilities u_j that the states E_j occur without reference to the initial states will be gained soluting the simultaneous the equations ;

$$\begin{cases} \sum_{\alpha} U_\alpha P_{\alpha j} = U_j \quad (\alpha, j=0, 1, 2\cdots) & \cdots(2) \\ \sum_{j} U_j = 1 & \cdots(3) \\ U_j \geqq 0 \end{cases}$$

The equation (3) always holds good if the number of the states E_n is finite, but, if infinite, (3) holds only if $\frac{b}{a}=\rho<1$, and all $u_j=0$ if $\rho\geqq 1$.

Now, the generating function k (z) of a probability distribution k_r is, from the equation (1),

$$K(z)=\sum_{r=0}^{\infty} K_r Z^r=\sum \frac{Z^r}{r!}\int_0^\infty e^{-\frac{v}{a}}\left(\frac{v}{a}\right)^r dB(v)=\int_0^\infty e^{-\frac{1-Z}{a}v}\ dB(v) \cdots\cdots(4)$$

Then as the Laplace transformation of the disposing time distribution $dB(v)$ is

$$B(s)=\int_0^\infty e^{-sv}\, dB\,(v) \cdots\cdots(5)$$

$$K(z)=\beta\left(\frac{1-z}{a}\right) \cdots\cdots(6)$$

Therefore, the function k (z) and the sequence of k_r's and consequently U_r's can be determined, as soon as $dB(v)$ has been specified. And it is better to employ such as $dB(v)$ and can easily be made its Laplace transform.

III) On the service-time distribution function B (v).

Kendall discribed in his papers especially about two cases of B (z);

(1) negative-exponential service-time,

$$dB(v)\equiv e^{-\frac{v}{b}}d\left(\frac{v}{b}\right) \cdots\cdots(7)$$

(2) constant service-time,

$$B(v)\equiv 0 \ (v<b); \quad B(v)\equiv 1 \ (v\geqq b) \cdots\cdots(8)$$

The most remarkable feature of these two distribution functions is that they have only one parameter b. But these are not very suitable for the actual cases like the disposingtime in a landing. Therefore, though it has two parameters, it should be better to use Γ-distribution in the χ^2 form

$$dB(v)\equiv ce^{-hv}\, v^{\nu-1}dv. \begin{cases} c=\frac{h^\nu}{\Gamma(\nu)},\ \nu>0,\ h>0 \\ 0< v <\infty \end{cases}$$

which was once employed by Erlang, that is seen in Kendall's papers. The equations (7) and (8) are the limiting forms of (9) when $\nu=1$ and $\nu\to\infty$ respectively.

The expectation and variance of the distribution (9) are respectively

$$E(v)=b=\frac{\nu}{h} \ and \ v(v)=\frac{\nu}{h^2} \cdots\cdots(10)$$

and its Laplace transformation is

$$\int_0^\infty e^{-sv}\, dB(v)=\frac{h^\nu}{(s+h)^\nu} \cdots\cdots(11)$$

The figures (1, 2, and 3) show the comparisons of the calculated values from the epuation (9) with the actual cases which was investigated in the Nagano district.

It is seen in these figures that the more the number of data is, the better the approximation is.

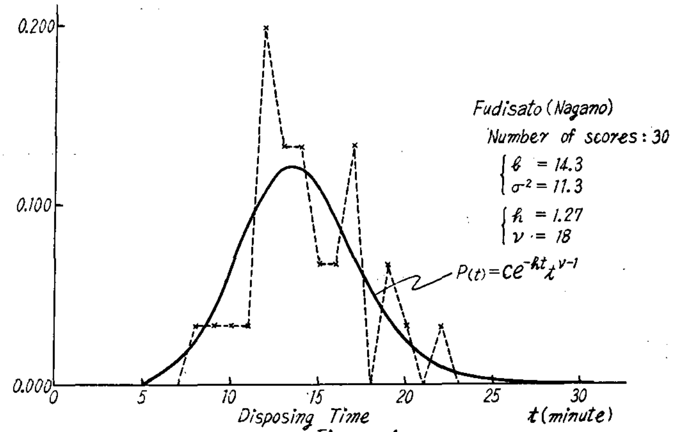

Figure 1

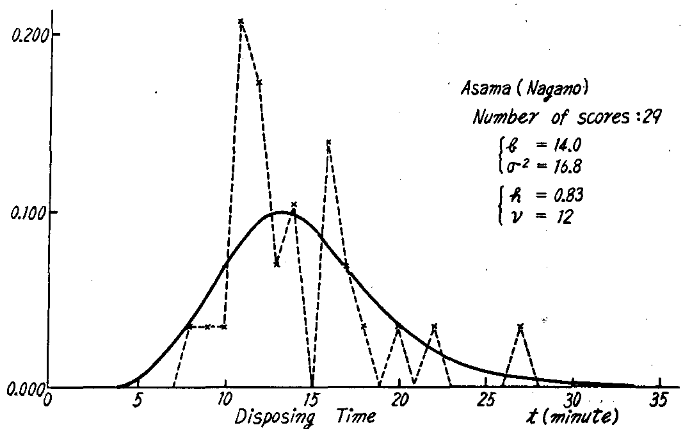

Figure 2

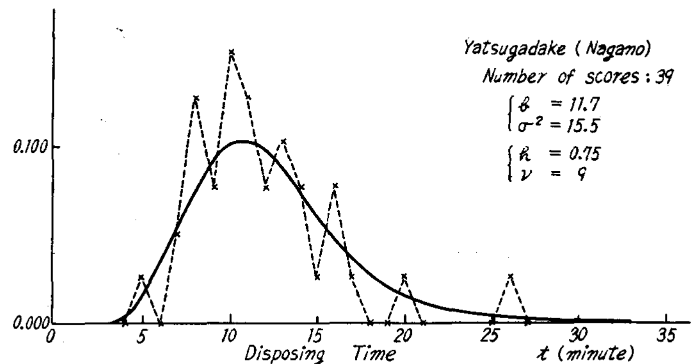

Figure 3

IV) The waiting of the workers in a landing.

It is now supposed that the landing is capable of accomodating $r+1$ loads of timbers, including the one being disposed, and that a tractor can not work any more as long as there are $r+1$ loads in the landing. Consequently, the number of possible states E_n at a regeneration point (when the disposal of each load has just been finished) is $r+1$; $n=0, 1, 2, 3, \ldots. r$, and any other states can not occur.

The formula will be as follows to compute the probabilities U_n of E_n (n=0, 1, 2, ...r) at a regeneration point, taking (9) as the disposing-time distribution.

From the equation (6) and (11),

$$K(z)=\beta\left(\frac{1-z}{a}\right)=\frac{h^{\nu}}{\left(\frac{1-z}{a}+h\right)^{\nu}} \quad \cdots\cdots (12)$$

By expanding the generating function k(z), equation (12), in powers of z,

$$K(z)=\frac{(ah)^{\nu}}{(l+ah)^{\nu}}\left\{1+\frac{\nu}{1+ah}z+\frac{\nu(\nu+1)}{2!(1+ah)^2}z^2+\cdots+\frac{\nu(\nu+1)\cdots(\nu+n-1)}{n!(1+ah)^n}z^n+\cdots\right\}$$

$$=k_0+k_1z+k_2z^2+\cdots\cdots, \quad \cdots\cdots (13)$$

Therefore,

$$\left.\begin{aligned} k_0&=\frac{(ah)^{\nu}}{(1+ah)^{\nu}} \\ k_n&=\frac{(ah)^{\nu}\,\nu(\nu+1)\cdots(\nu+n-1)}{n!(1+ah)^{\nu+n}} \end{aligned}\right\} \quad \cdots\cdots (14)$$

$$(n=1, 2\cdots\cdots\cdots\cdots)$$

With the equation (14), the transition matrix (p) is

$$\begin{array}{c|cccc} \nearrow & 0 & 1 & 2\cdots\cdots r & \\ \hline 0 & k_0 & k_1 & k_2 \cdots k'_r & \\ 1 & k_0 & k_1 & k_2 \cdots k'_r & \\ 2 & 0 & k_0 & k_1 \cdots k'_{r-1} & \\ \vdots & \vdots & \vdots & \vdots \quad \vdots & \\ r & 0 & 0 & \cdots\cdots k'_1 & \end{array} \qquad k'_n=1-\sum_{j=0}^{n-1}k_j \quad \cdots\cdots (15)$$

Here, U_j's (j=0, 1,....r) are now able to be computed, with the equation (2), (3), and (15). U_o is the probability of the waiting of the workers in the landing.

V) The waiting of a tractor.

Here, it will also be supposed the same condition as in IV) about the width of landing.

The U_n's gained in the section IV) are the probabilities of E_n's occurring at epochs when each disposing-time has just been finished (at the generation points). So the probability of the waiting states of a tractor (or the state that the landing is filled with timber) E_{r+1} cannot be gained by the method above mentioned. And E_n only indicates that the waiting state of tractor has just ended.

Then, how can we get the waiting time of a tractor? On my opinion, it must be convenient to compute it from equation (16) supposing that the amounts of imput and disposed timbers during a certain fairly long time are almost equal.

$$\frac{T}{b}(1-U_0)W=\frac{T}{a}(1-U_T)W$$

therefore,

$$U_T=1-\frac{a}{b}(1-U_0) \quad \cdots\cdots (16)$$

here,

T : a time

W : mean volume of a load hauled by a tractor a : expectation of the intervals of the consecutive arrivals

b : mean disposing time (service time) of a load

U_T : probability caused by the tractor waiting

On the other hand, considering it theoretically, the computing equation is as follows.

By the condition that the imput is Poissonian $P_r=\frac{e^{-\lambda t}(\lambda t)^r}{r!}$, when the attension is only directed to the regeneration point at which E_{r-n} appears, the waiting time of a tractor during the next disposing time v of a load is

$$\int_0^v \left[1-e^{-\lambda t}\left\{1+\lambda t+\cdots\cdots+\frac{(\lambda t)^n}{n!}\right\}\right](v-t)dt \cdots\cdots (17)$$

Therefore, the rate of waiting of a tractor (the probability of waiting) during the same time is

$$R_{r-n}=\int_0^\infty \frac{1}{v}\left[1-e^{-\lambda t}\left\{1+\lambda t+\cdots\cdots+\frac{(\lambda t)^n}{n!}\right\}\right](v-t)\,dt\,dB(v) \cdots\cdots (18)$$

Here, we can get U_T (the probability of tractor waiting without reference to the initial condition) using the equation (18) and U_j ($j=0, 1, \ldots .r$) gained above.

$$U_T=\sum_{n=0}^{r} U_{r-n}\ R_{r-n}$$

VI) miscellany

What is above mentioned stands on the assumption that a tractor arrives at a landing at random (Poissonian imput). But actually there are often the cases where the assumption is not correct. For instance, when only one tractor works, it is naturally supposed that the intervals of consecutive arrivals have the distribution in the X^2 form, although, if many tractors work, the assumption must nearly be correct.

However, these problems have not yet been discussed thoroughly so far.

要　　約

トラクター集材作業においては，集材土場において土場作業員とトラクター作業員との間に作業手待ちを生ずる。

今土場の広さが $r+1$ 荷分（トラクター1台が1回に集材する材の量を1荷と呼ぶ）であるとする。そして，土場が，n荷分だけ未処理材でつまつている状態を E_n であらわし，その状態の起る確率を P_n であらわせば，P_0 は，土場作業員の手待ちの起る確率をあらわす。

トラクター到着間隔がランダムと考えられるときには，P_n ($n=0.1\cdots\cdots r$) は，次の連立方程式によつて計算できる。

$$\begin{cases} AP=P & \cdots\cdots (2) \\ \sum_{j=0}^{r} P_j=1 & \cdots\cdots (3) \end{cases}$$

ここに

$$P=\begin{pmatrix} P_0 \\ P_1 \\ \vdots \\ P_r \end{pmatrix} \qquad A=\begin{pmatrix} k_0 & k_0 & 0 & 0\cdots 0 \\ k_1 & k_1 & k_0 & 0\cdots 0 \\ \vdots & \vdots & \vdots & \vdots \quad \vdots \\ k_r' & k_r' & k'_{r-1} & \cdots k_1' \end{pmatrix}$$

$$k_0=\frac{(ah)^\nu}{(1+ah)^\nu}$$

$$k_n=\frac{(ah)^{\nu}\,\nu(\nu+1)\cdots(\nu+n-1)}{n!\,(1+ah)^{\nu+n}}\quad(n=1,\cdots r-1)$$

ここに，トラクター到着時間間隔の平均値をaとした。また土場における1荷処理時間の平均を $E_{(t)}$，分散 $V_{(t)}$ とすれば

$$E_{(t)}=\frac{\nu}{h}\qquad V_{(t)}=\frac{\nu}{h^2}$$

である。

なお，トラクター手待ち確率（P_{r+1}）は，P_0 と $E_{(t)}$ と a を用いて本文（17）式から間接に計算できる。